

Foundations of Language Science and Technology

Semantics 2

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Truth, Satisfaction, Entailment



- A formula A is **true** in model structure M
iff $[[A]]^{M,g} = 1$ for every variable assignment g .
- A formula A is **valid** ($\models A$)
 - iff A is true in every model structure.
- A formula A is **satisfiable** iff there is a model structure M in which A is true
- A set of formulas Γ is (simultaneously) **satisfiable** iff there is a model structure M in which all $A \in \Gamma$ are true (we also say that M simultaneously satisfies Γ , or M is a model of Γ).
- A set of formulae Γ is **contradictory** iff it is not satisfiable.
- A set of formulas Γ **entails** formula A ($\Gamma \models A$) iff A is true in every model structure that satisfies Γ .

Overview



- Semantic Processing - Introduction
- Logic-based meaning representation and processing:
Truth-conditional interpretation, entailment, deduction
 - First-order predicate as a representation language
 - Truth-conditional interpretation
 - The logical entailment concept
 - Deduction systems and theorem provers
- Word Meaning: Lexical-semantic resources, ontologies, similarity-based approaches
- Semantic Composition: Composing sentence and text meaning from word meaning
- Textual Entailment and Inference

Truth-conditional entailment checking



$\{A \rightarrow B, \neg B\} \models \neg A$?

	A	B	$A \rightarrow B$	$\neg B$	$\neg A$
M_1	1	1	1	0	0
M_2	1	0	0	1	0
M_3	0	1	1	0	1
M_4	0	0	1	1	1

Truth-conditional entailment checking



- $\forall d (\text{dolphin}(d) \rightarrow \text{mammal}(d)), \text{dolphin}(\text{flipper}) \models \text{mammal}(\text{flipper})$?
- Computing entailment and other logical concepts through semantic interpretation is inefficient and in many cases infeasible.
- A strictly syntactic way of checking validity, satisfiability, and entailment, using rewrite of logical formulae only, is provided by .
[deduction calculi](#) (or [proof theoretic systems](#)).

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Axioms and Deduction Rules



- Deduction calculi are typically made up of
 - (1) axioms (which can be unconditionally used in every proof)
 - (2) deduction rules
- Example for a frequently used axiom:
 - $\text{Av } \neg A$ ("Tertium non datur")
- Example for a frequently used deduction rule ("Modus Ponens")

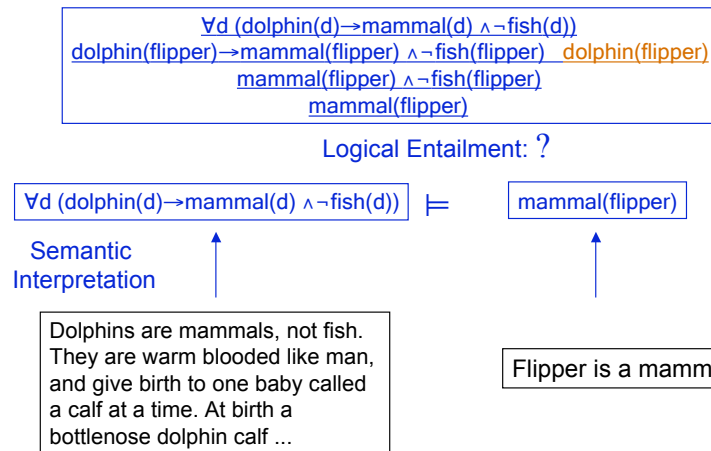
$$\frac{A \rightarrow B, A}{B}$$

Derivation / Derivability



- Derivation and Derivability in a Hilbert-style deduction calculus:
- Formula A is derivable (deducible) from a set of formulas Γ ($\Gamma \vdash A$) iff there is a derivation with premisses Γ and conclusion A .
- A derivation of A from premisses Γ is a sequence of formulas A_1, \dots, A_n such that $A_n = A$, and for all members A_i of the sequence: either
 - A_i is an (instantiation of an) axiom, or
 - $A_i \in \Gamma$, or
 - A_i is the result of the application of a deduction rule, whose conclusion is A_i , and whose premisses all occur in the sequence before A_i

Is Flipper a mammal?



A simple derivation example



(1) $\forall d (\text{dolphin}(d) \rightarrow \text{mammal}(d) \wedge \neg \text{fish}(d))$	Premiss
(2) $\text{dolphin}(\text{flipper}) \rightarrow \text{mammal}(\text{flipper}) \wedge \neg \text{fish}(\text{flipper})$	Universal Instantiation: $\forall x A \vdash A[x/a]$
(3) $\text{dolphin}(\text{flipper})$	Premiss
(4) $\text{mammal}(\text{flipper}) \wedge \neg \text{fish}(\text{flipper})$	Modus Ponens: $A, A \rightarrow B \vdash B$ (2), (3)
(5) $\text{mammal}(\text{flipper})$	Conjunction reduction (4)

which proves that

$\forall d (\text{dolphin}(d) \rightarrow \text{mammal}(d)), \text{dolphin}(\text{flipper}) \vdash \text{mammal}(\text{flipper})$

But what about entailment?

$\forall d (\text{dolphin}(d) \rightarrow \text{mammal}(d)), \text{dolphin}(\text{flipper}) \models \text{mammal}(\text{flipper})$?

Central proof-theoretic concepts



- Formula A is derivable (deducible) from a set of formulas Γ ($\Gamma \vdash A$) in a given deduction system, iff one can obtain A starting from Γ , by using deduction rules and possibly axioms of that deduction system.
- A formula A is provable ($\vdash A$) iff $\emptyset \vdash A$
- A set of formulas Γ is inconsistent iff there is a formula A such that $\Gamma \vdash A$ and $\Gamma \vdash \neg A$. Otherwise, it is consistent.

Semantic Tableaux



- Semantic tableau calculus: Derivation and proofs by generation of tableau trees via decomposition rules.
- $\{A, B\}$ reads: add A and B to the tableau.
- $\{A\}, \{B\}$ reads: Split tableau, and add A and B , respectively, to right and left branch

	Affirmative context	negative context
$A \wedge B$	$\{A, B\}$	$\{\neg A\}, \{\neg B\}$
$A \vee B$	$\{A\}, \{B\}$	$\{\neg A, \neg B\}$
$A \rightarrow B$	$\{\neg A\}, \{B\}$	$\{A, \neg B\}$
$A \leftrightarrow B$	$\{A \rightarrow B, B \rightarrow A\}$	$\{\neg(A \rightarrow B)\}, \{\neg(B \rightarrow A)\}$
$\forall x A$	$A[a/x]$ for arbitrary a	$\neg A[a/x]$ for a new a
$\exists x A$	$A[a/x]$ for a new a	$\neg A[a/x]$ for arbitrary a

Semantic Tableau Calculus



- To prove A from premisses Γ , add its negation and show that the result is inconsistent.
- A subtableau is closed, iff it contains A and $\neg A$
- A tableau is closed iff all subtableaus are closed.
- To prove A from premisses Γ , add its negation and show that the result is inconsistent:
 $\Gamma \vdash A$ iff the tableau for $\Gamma \cup \{\neg A\}$ is closed
This kind of proof is called a refutation proof.

Deduction Calculi



- There is one model-theoretic interpretation (for standard predicate logic).
- There is a wide variety of deduction calculi, e.g.:
 - Hilbert calculus
 - Semantic tableau calculus
 - Calculus of natural deduction (Gentzen calculus)
 - Resolution-based systems
- Logical deduction calculi are useful only in as far their derivability and provability concepts (\vdash) mirror / are co-extensional with the truth-conditionally based concepts of entailment and validity (\models).

Soundness and Completeness



- We call a deduction system **sound**, if we can derive only semantically entailed formulae from a set of premisses: No derivation of a false hypothesis from a true text!
- We call a deduction system **complete**, if it provides derivations for all entailed formulae from a set of premisses.
- **Soundness**: If $\Gamma \vdash A$, then $\Gamma \models A$.
- **Completeness**: If $\Gamma \models A$, then $\Gamma \vdash A$.
- Soundness and completeness have been proven for many different deduction systems.
- For sound and complete deduction systems, the proof-theoretic concepts coincide with the corresponding semantic ones:

Validity	Provability
Entailment	Derivability/Deducibility
Satisfiability	Consistency

Efficiency Matters



- Propositional (quantifier-free) logic is NP-complete (it requires exponential time in dependence of the number of clauses)
- FOL is undecidable (provable / valid formulas are recursively enumerable)
- Implemented deduction systems - originally mostly developed for purposes of mathematical theorem proving - allow very efficient derivability / entailment checks.
- Theorem provers for sub-languages of FOL (horn-clause logic, description logics) are yet more efficient.



We distinguish:

- **Theorem provers** (in the narrow sense) typically with
 - Refutation method
 - **Resolution** proof procedure
 - Input: Set of formulas (premisses)
 - Output: Yes, if proof successful
 - Examples: Vampire, SPASS, BLIKSEM, OTTER
- **Model generators**
 - Check consistency
 - Using tableau techniques
 - Output is Yes, if the hypothesis is consistent with the premisses
 - Plus a model for $\Gamma \cup \{A\}$ as an important side effect.
 - Examples: MACE, KIMBA

Lexical semantics for information access



T: Aki Kaurismäki directed his first full-time feature

H: Aki Kaurismäki directed a film

T: His wife Strida won a seat in parliament after forging an alliance with the main anti-Syrian coalition in the recent election.

H: Strida elected to parliament.

T: Oscar-winning actor **Nicolas Cage** and Superman have sth. in common

H: Nicolas Cage was awarded an Oscar.

T: Wyniemko, now 54 and living in Rochester Hills, was arrested and tried in 1994 for a rape in Clinton Township.

H: Wyniemko was accused of rape.



- What we have:
 - A highly precise and efficient methods to derive certain kinds of information from text documents.
- What we need:
 - Lexical semantic information
 - A mapping from text to (contextually appropriate) FOL representations
 - Extra-linguistic knowledge supporting entailment

Dolphins in First-order Logic



Dolphins are mammals, not fish.

$\forall d (\text{dolphin}(d) \rightarrow \text{mammal}(d) \wedge \neg \text{fish}(d))$

Dolphins live-in pods.

$\forall d (\text{dolphin}(d) \rightarrow \exists x (\text{pod}(p) \wedge \text{live-in}(d,p)))$

Dolphins give birth to one baby at a time.

$\forall d (\text{dolphin}(d) \rightarrow \forall x \forall y \forall t (\text{give-birth-to}(d,x,t) \wedge \text{give-birth-to}(d,y,t) \rightarrow x=y)$

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Content words



Dolphins are mammals, not fish. They are warm blooded like man, and give birth to one baby called a calf at a time. At birth a bottlenose dolphin calf is about 90-130 cms long and will grow to approx. 4 metres, living up to 40 years. They are highly sociable animals, living in pods which are fairly fluid, with dolphins from other pods interacting with each other from time to time.

The dolphin text

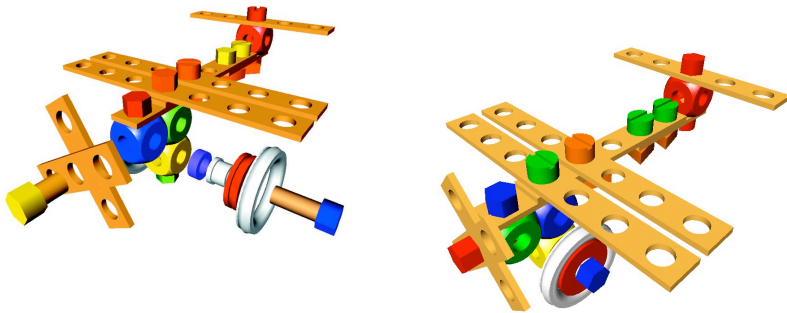


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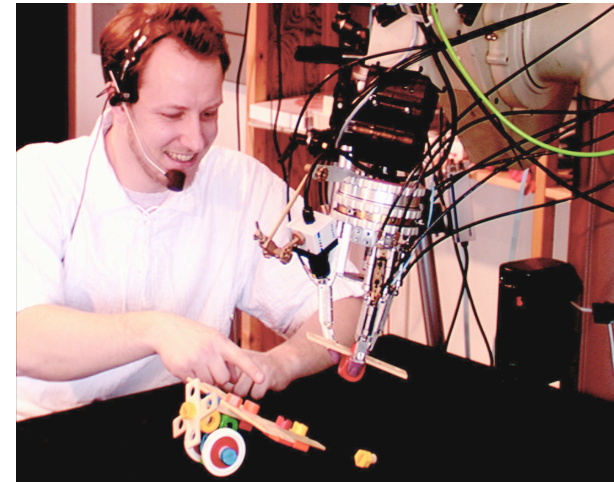
Word-meaning is multi-layered



A robotics application



Collaborative Research Center
„Artificial Situated Communicators“
Bielefeld



Diversity of word meaning



- The concepts corresponding to single readings of a word are typically multi-layered, consisting of heterogeneous kinds of information (crossing modality), among other things:
 - **Propositional** or conceptual information - can be defined or paraphrased in language, represented in a logical or terminological framework („ontology“)
 - Visual (or other sensory) **prototypical** information
 - Stereotypical information - valid in the „normal“, default case
- No clear-cut boundary between word meaning and world knowledge.
- No clear-cut boundary between common-sense meaning and domain-specific information (usually provided by „domain ontologies“)

Diversity of Word Meaning



- There is no chance to come up with a lexical-semantic repository which provides full information about word meaning.
- Lexical meaning description can only be partial (restricted to one semantic layer).
- Lexical meaning description should be guided by the needs of a certain type of task.
- The task/ semantic layer which has been mainly focussed on in computational linguistics, and which we will consider in more detail are propositional/ conceptual knowledge for information access in text databases.



- An ontology is a shared conceptualization of a domain
- An ontology is a set of definitions in a formal language for terms describing the world
(Definition taken from slides of Adam Pease)
- Another definition: Ontologies are
 - Hierarchical data structures
 - Providing formally rigorous information about concepts and relation
 - Within a specific domain ([domain ontologies](#))
 - Or concepts and relations of foundational, domain-independent relevance ([upper ontologies](#))
- Upper Ontologies:
 - DOLCE, CYC, SUMO
- WordNet is a linguistically motivated and language related upper ontology, therefore sometimes called a „language ontology“.